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Preliminary identification and characterization of nonlinear wave-wave, wave-beam, and wave-particle interactions in beam-driven tokamak plasma

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Abstract

We present the analysis of 18 DIII-D shots heated by neutral-beam injection (NBI). [Typical plasma parameters are n~$2x10^{13}$ cm^{-3}, T~1keV.] As energetic ions seeded by NBI resonate at the frequencies of various Alfven eigenmodes (AEs), we observe rich toroidal Alfven eigenmode (TAE) activity as these weakly-damped modes are driven. Both steady and modulated beam power have been investigated.

Statistical techniques were used to benignly filter out the polluting effect of edge-localized modes (ELMs) in the observed fluctuation spectra. Guided by recent simulations [*Spong et al 2021; Nucl. Fusion 61, 116061*] which identify nonlinear coupling between TAEs and zonal flows, we seek to correlate the observed nonlinear interaction between ensembles of AEs and lower-frequency MHD modes with coincident perturbations in the fast-ion distribution function.

Evidence for energy exchange due to this coupling is given by higher-order spectral techniques. In particular, we report on the evolution of consistent bicoherent features in magnetic fluctuation data.

Introduction to TAEs

Toroidal geometry facilitates eigenmodes[1]

Alfven eigenmodes (AEs) are an interaction of shear Alfven waves (SAW),

$$\omega_A(r) = k_{||}(r) v_A(r)$$

Parallel wave-vector is given approximately by

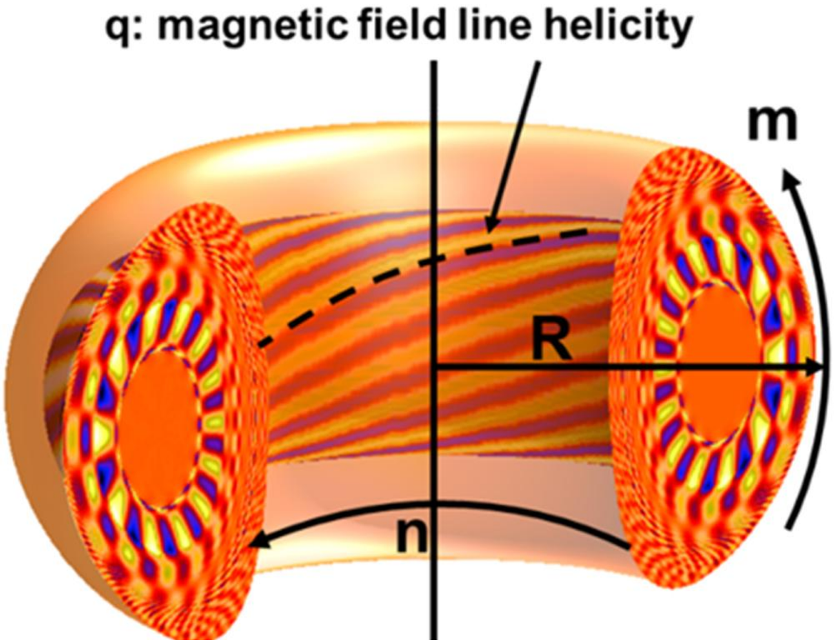
$$k_{||m}(r) = \frac{1}{R}\left(n - \frac{m}{q(r)}\right)$$

Toroidal Alfven eigenmode (TAE) results from counter-propagation

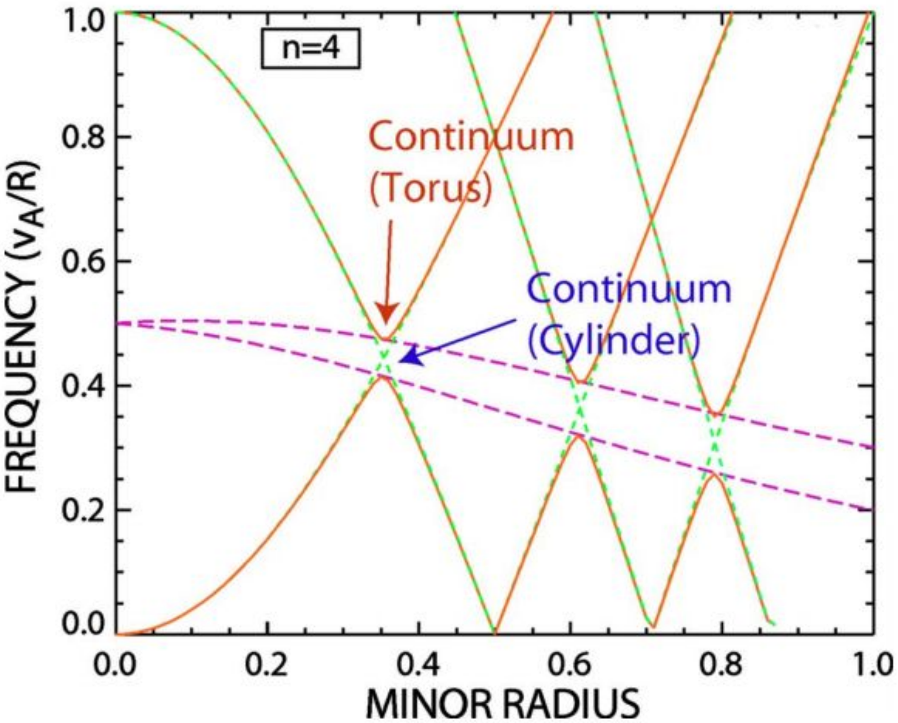
$$k_{||m}(r_0) = -k_{||m+1}(r_0)$$

[1] Pinches, S., *Nonlinear Interaction of Fast Particles with Alfven Waves in Tokamaks*, University of Nottingham (1996)

Interaction of counter-propagating cylindrical modes provides frequency gap[2]



Garcia-Muñoz, M.et al.2019 Active control of Alfvén eigenmodes in magnetically confined toroidal plasmas. Plasma Phys. Control. Fusion 61, 054007



[2] Heidbrink, W., *Basic physics of Alfvén instabilities…*, Physics of Plasmas 15, 055501 (2008); https://doi.org/10.1063/1.2838239

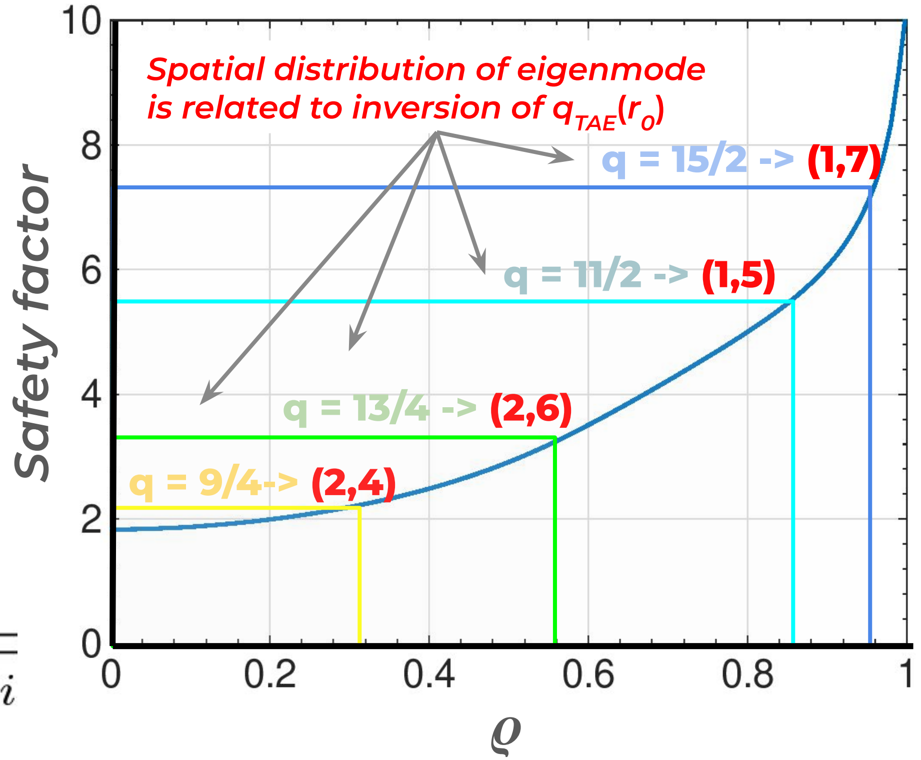
Safety factor profile determines TAE localization[1]

Critical value of safety factor is

$$q_{\mathrm{TAE}}(r_0) = \frac{m + 1/2}{n}$$

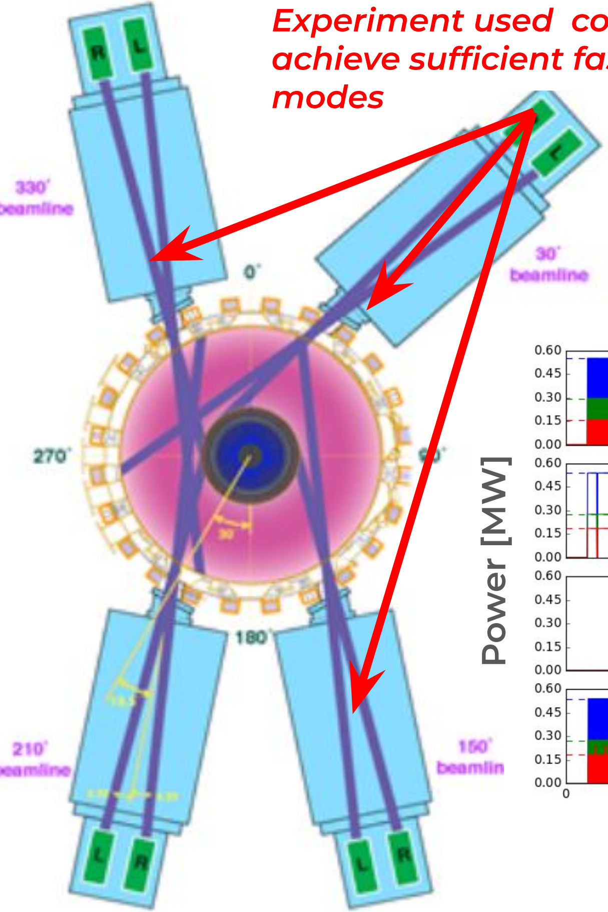
TAE frequency (in local frame) is

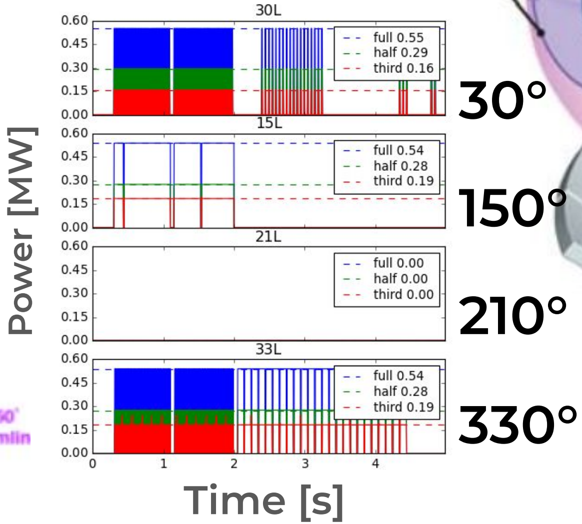
$$f_{\mathrm{TAE}} = \frac{v_A}{4\pi q_{\mathrm{TAE}} R}$$
$$\approx \frac{1}{(4\pi)^{3/2}} \frac{nB}{R(m+1/2)\sqrt{m_i n_i}}$$

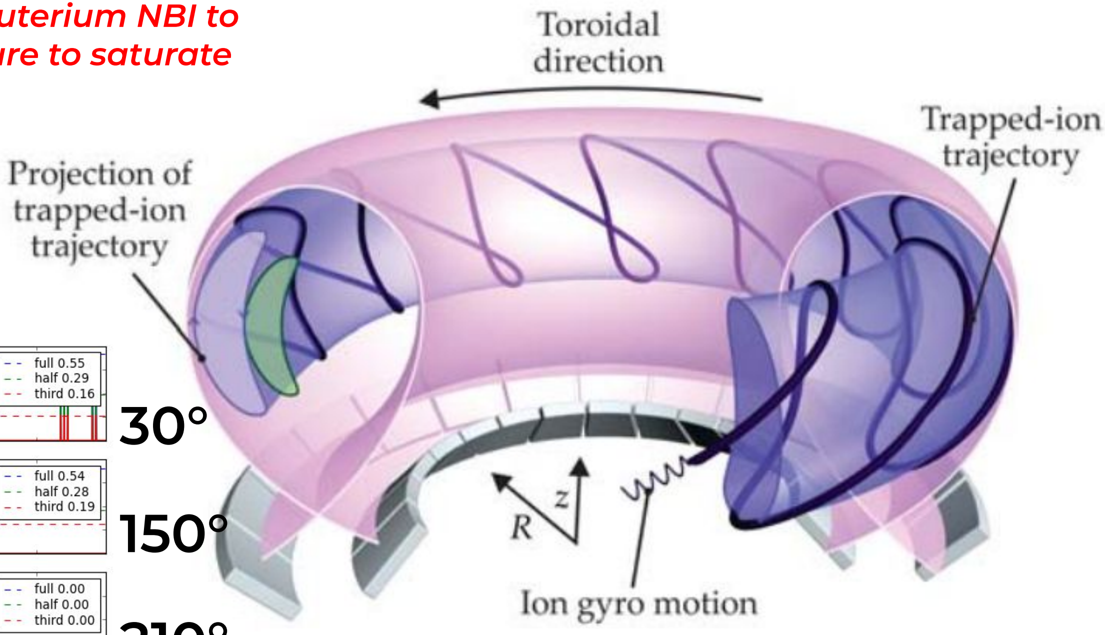


NBI seeds fast-ion population which drives TAEs[2]

Experiment used co-injected deuterium NBI to achieve sufficient fast-ion pressure to saturate modes







$$\omega + (m+l)\omega_\theta - n\omega_\zeta \simeq 0$$

Pace, D.., Heidbrink, W., and Van Zeeland, M 2015 *Keeping fusion plasmas hot*. Physics Today 68.10 https://doi.org/10.1063/PT.3.2946

Pawley, C.J. et al. 2017 *Advanced control of neutral beam injected power in DIII-D*. Fusion Engineering and Design. 123, 453-7.

Experimental data

Both steady and modulated beam power investigated

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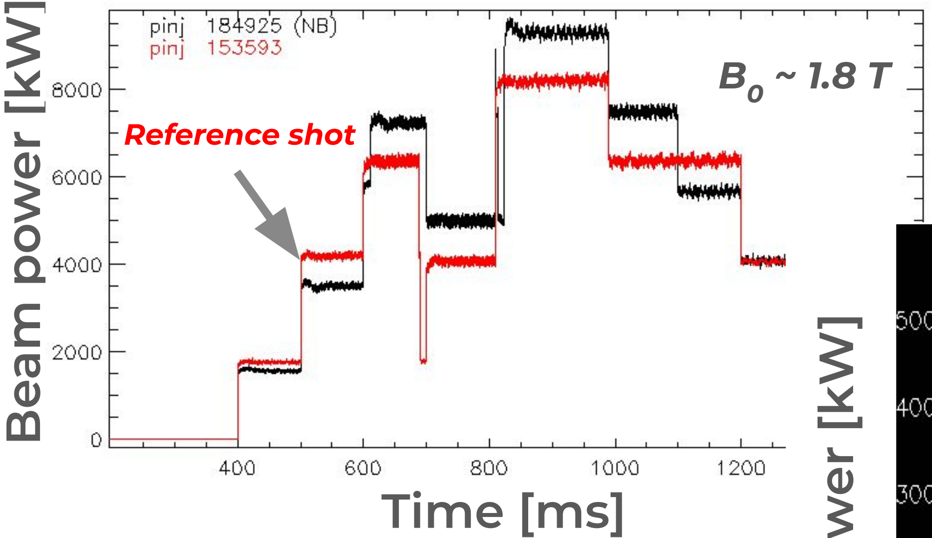

Favorable discharge parameters informed via investigation of archived data-set[3]

DIII-D platform delivers high reproducibility of plasma conditions and NBI programming

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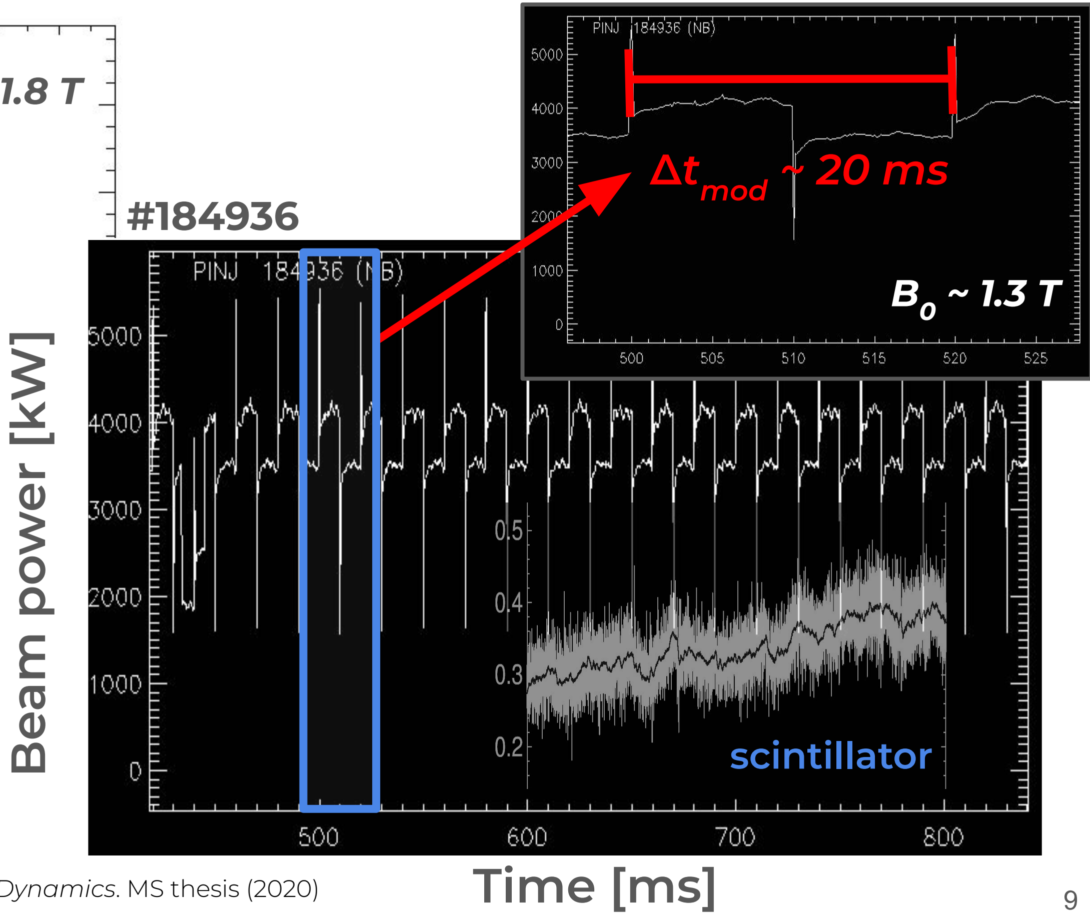
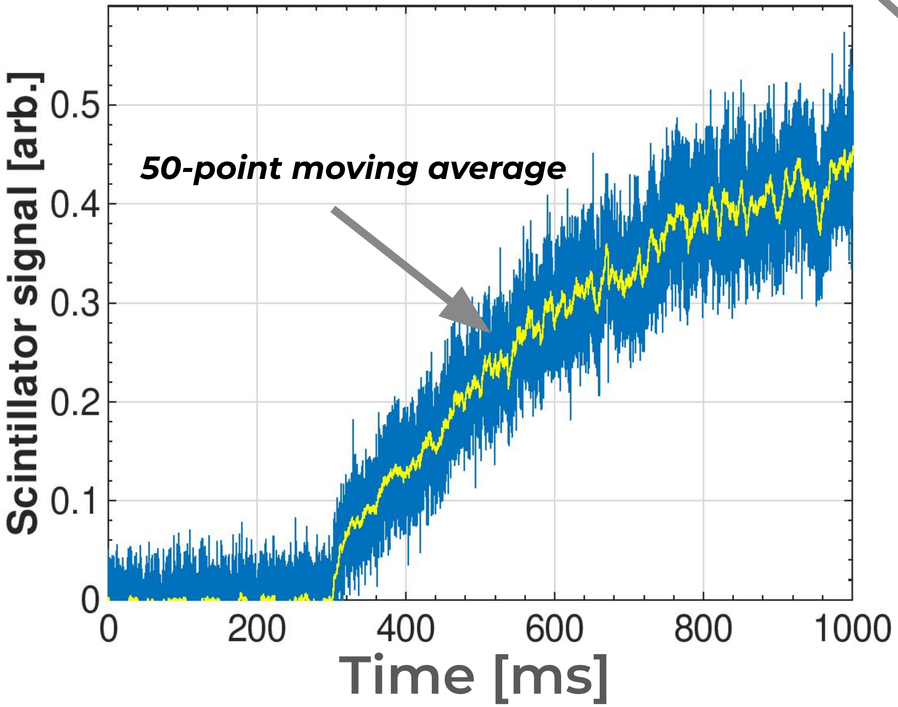
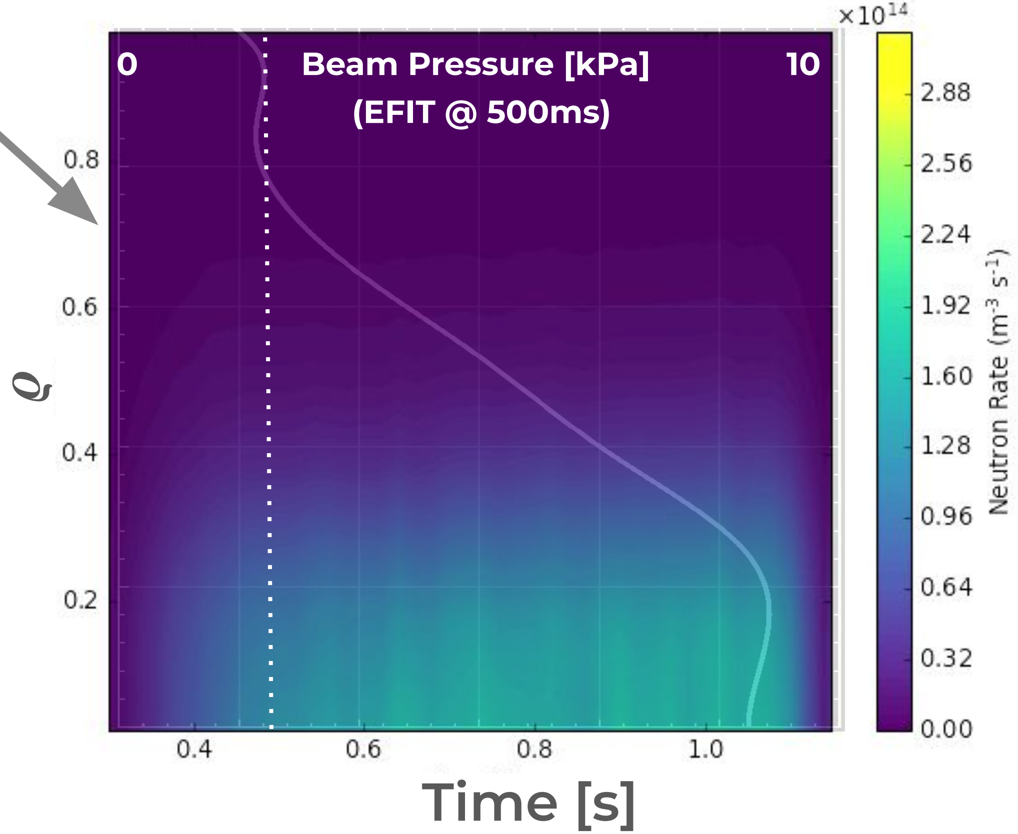


[3] Riggs, G., *Interpretations of Bicoherence in Space & Lab Plasma Dynamics*. MS thesis (2020) https://doi.org/10.33915/etd.7655.

Deposition of beam energy facilitates fusion reactions, enhancing neutron flux

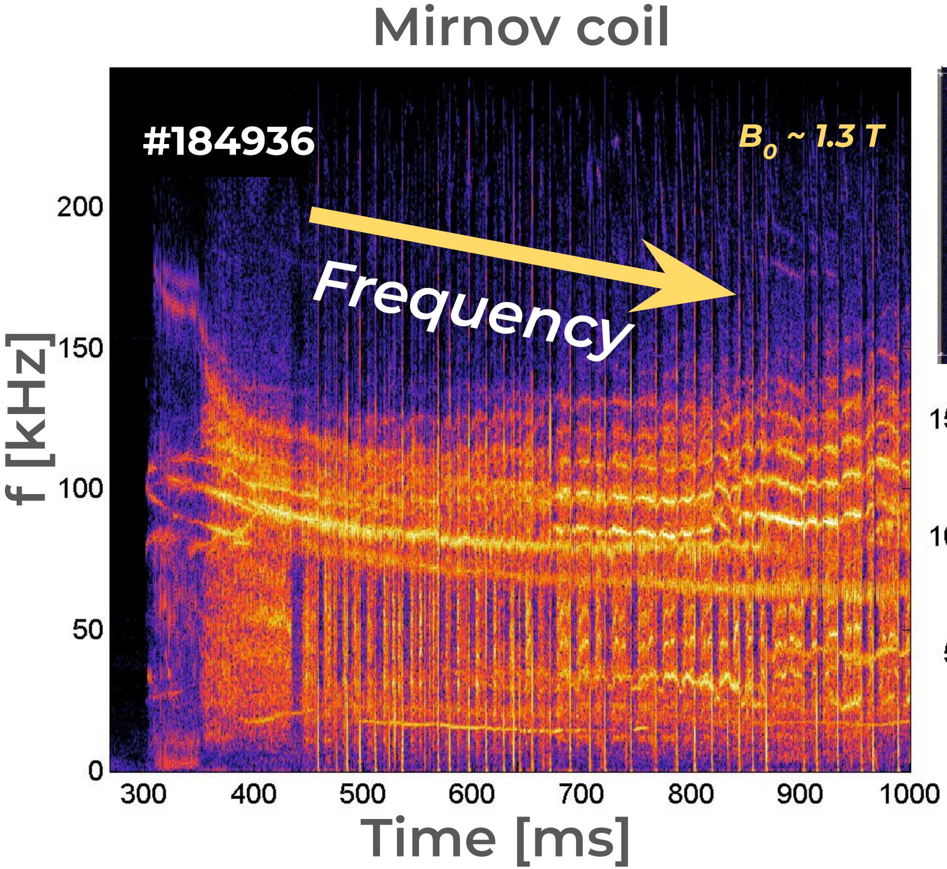
Scintillator data compares favorably with output of RABBIT[4] simulation

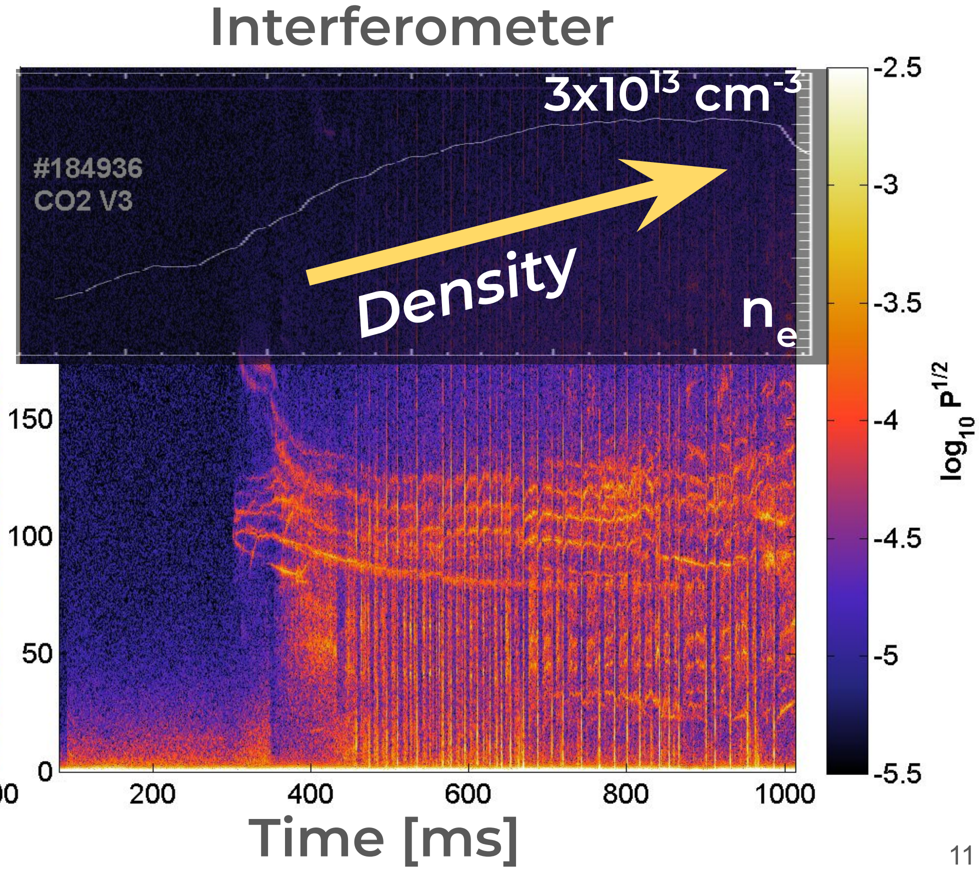




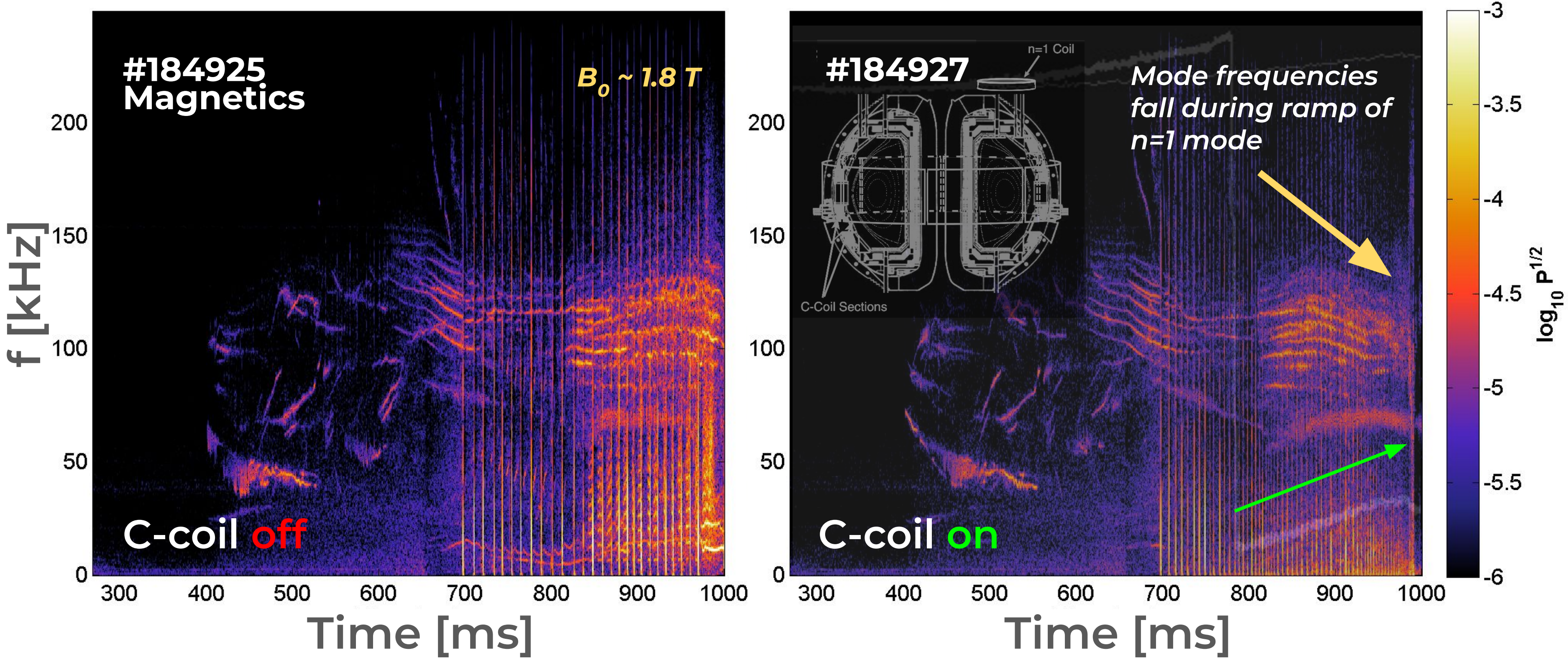
[4] M. Weiland et al., *RABBIT: Real-time simulation of the NBI fast-ion...*, Nucl. Fusion 58 082032 (2018); https://doi.org/10.1088/1741-4326/aabf0f

Density ramp interrogates the mechanism of TAE coupling via scan of frequency & amplitude



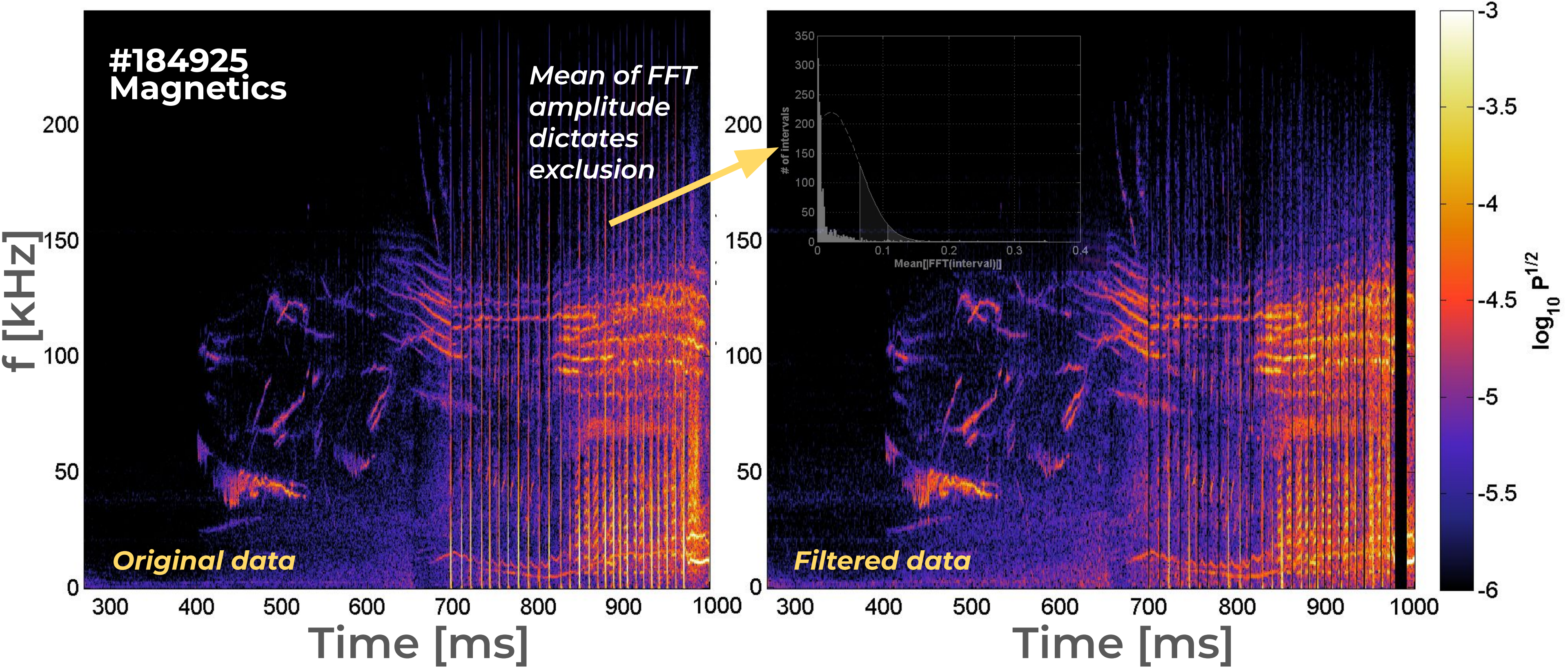


Eigenmodes augmented by C-coil perturbation

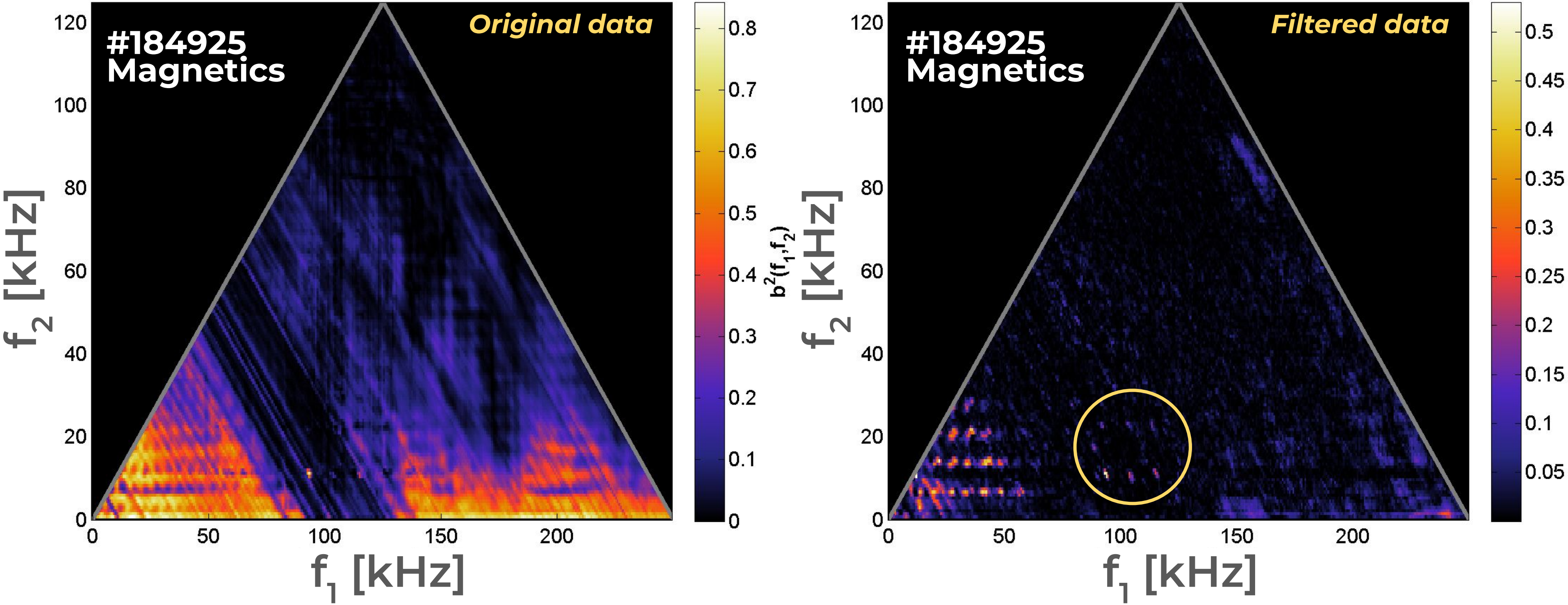


Wave-wave interaction

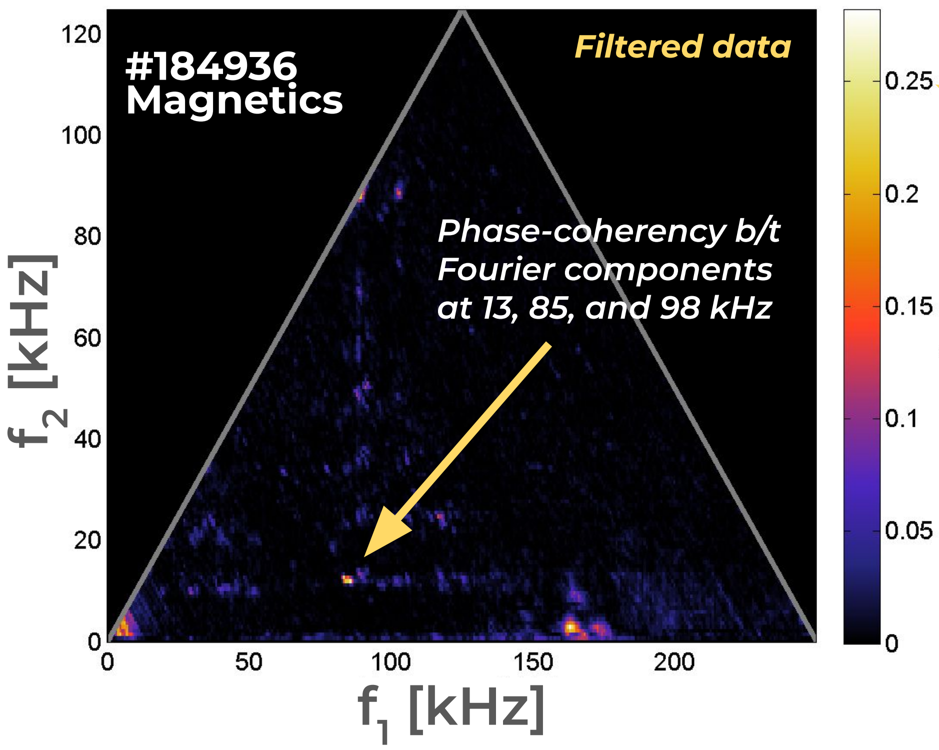
Acquired spectrograms are ELM-filtered conveniently

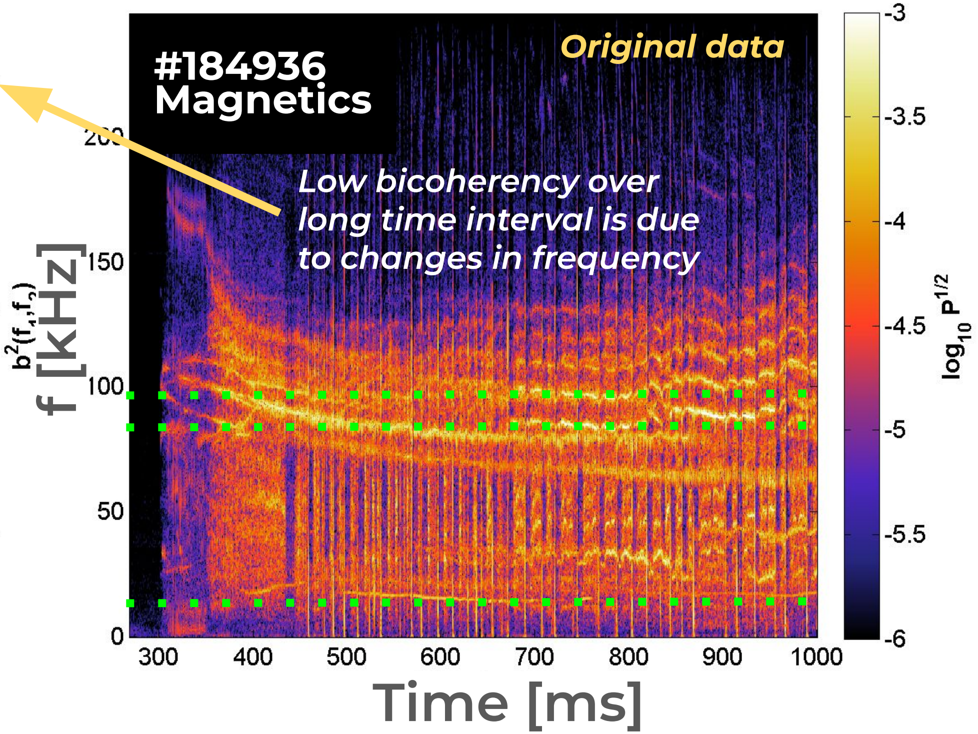


ELM-filtered bispectra reveal signatures of 3-wave interaction

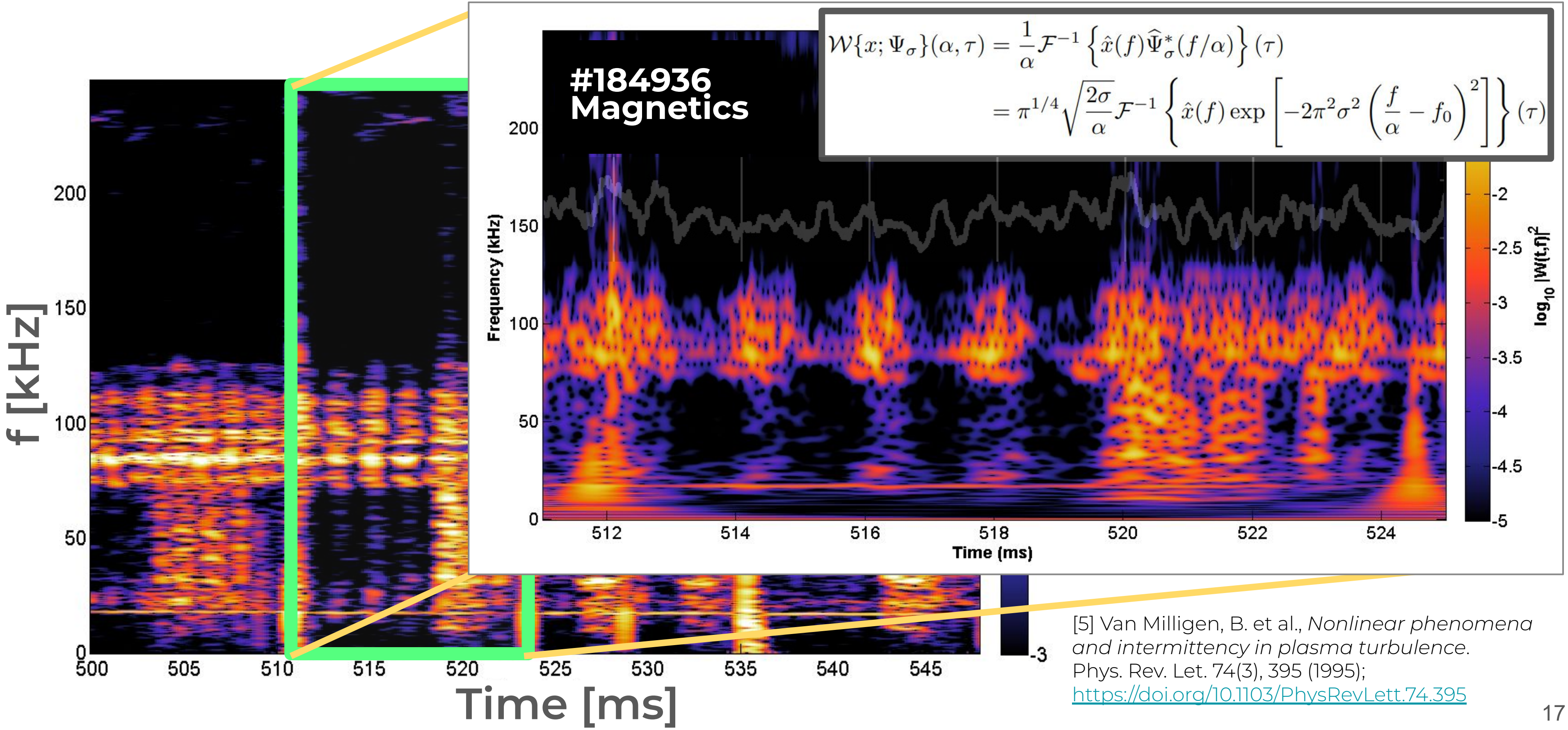


Nonlinear wave-wave coupling inferred from magnetics data





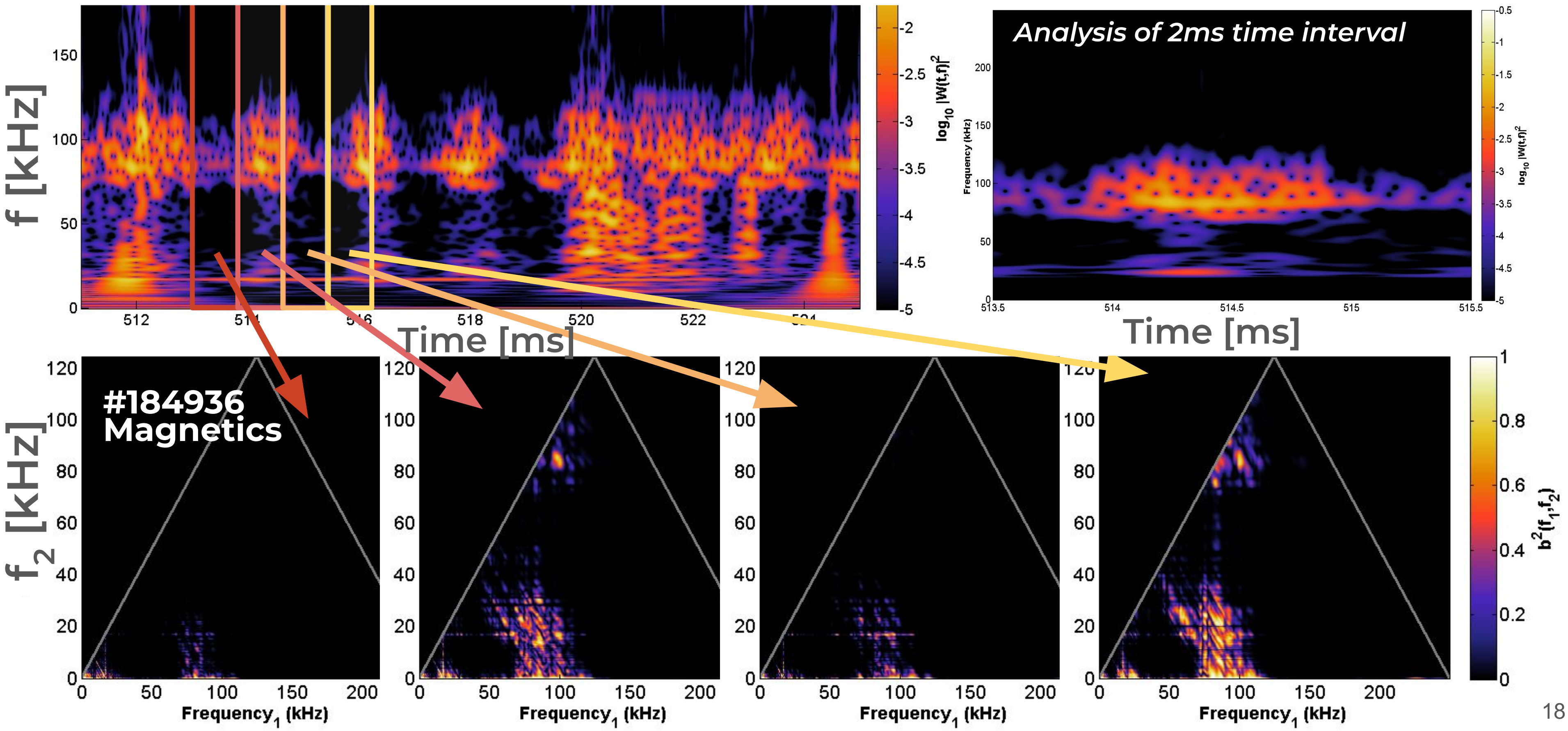
Morlet wavelet optimizes simultaneous resolution in time and frequency[5]



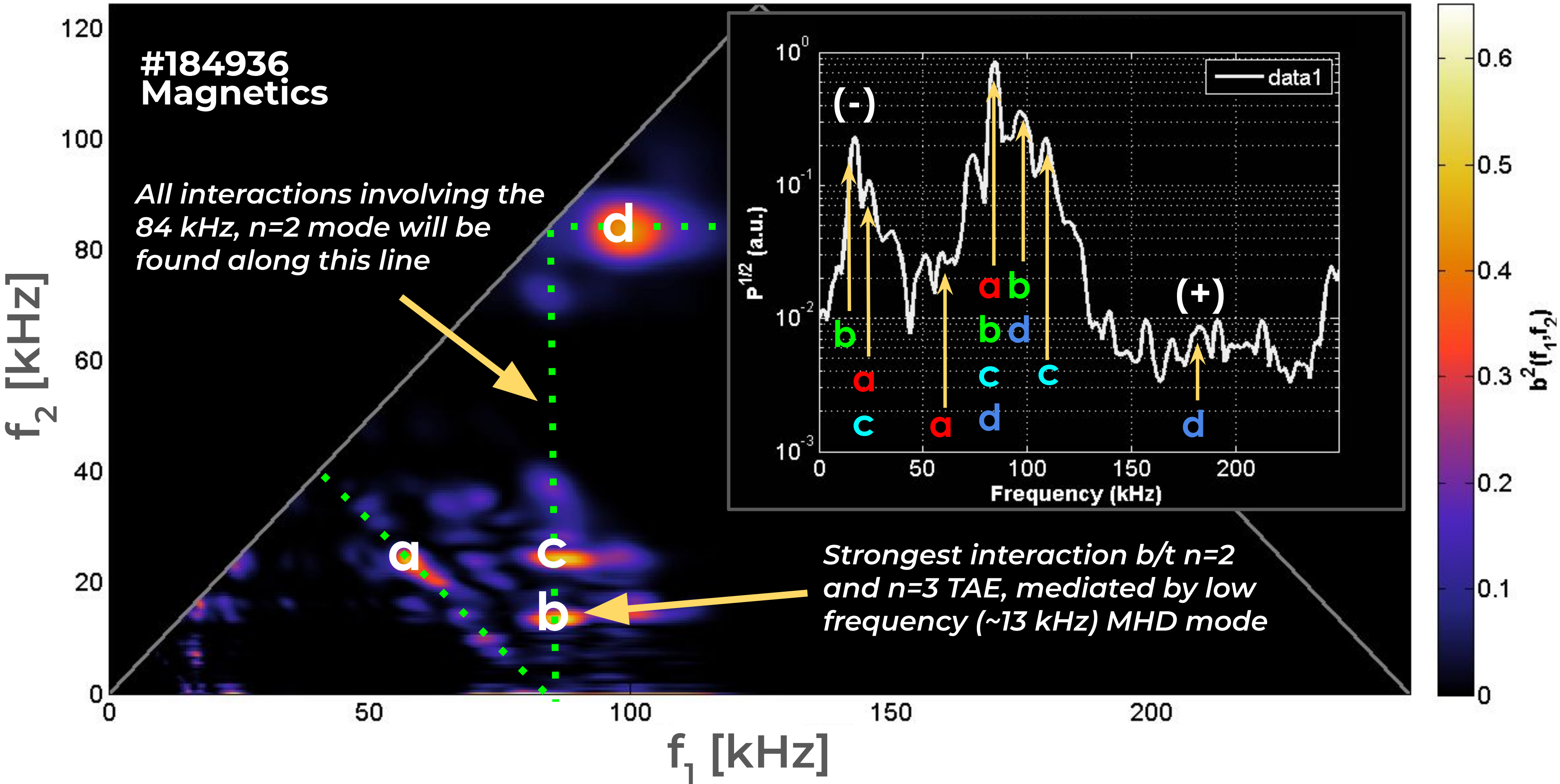
$$\mathcal{W}\{x;\Psi_\sigma\}(\alpha,\tau) = \frac{1}{\alpha}\mathcal{F}^{-1}\left\{\hat{x}(f)\widehat{\Psi}^*_\sigma(f/\alpha)\right\}(\tau)$$
$$= \pi^{1/4}\sqrt{\frac{2\sigma}{\alpha}}\mathcal{F}^{-1}\left\{\hat{x}(f)\exp\left[-2\pi^2\sigma^2\left(\frac{f}{\alpha}-f_0\right)^2\right]\right\}(\tau)$$

[5] Van Milligen, B. et al., *Nonlinear phenomena and intermittency in plasma turbulence.* Phys. Rev. Let. 74(3), 395 (1995); https://doi.org/10.1103/PhysRevLett.74.395

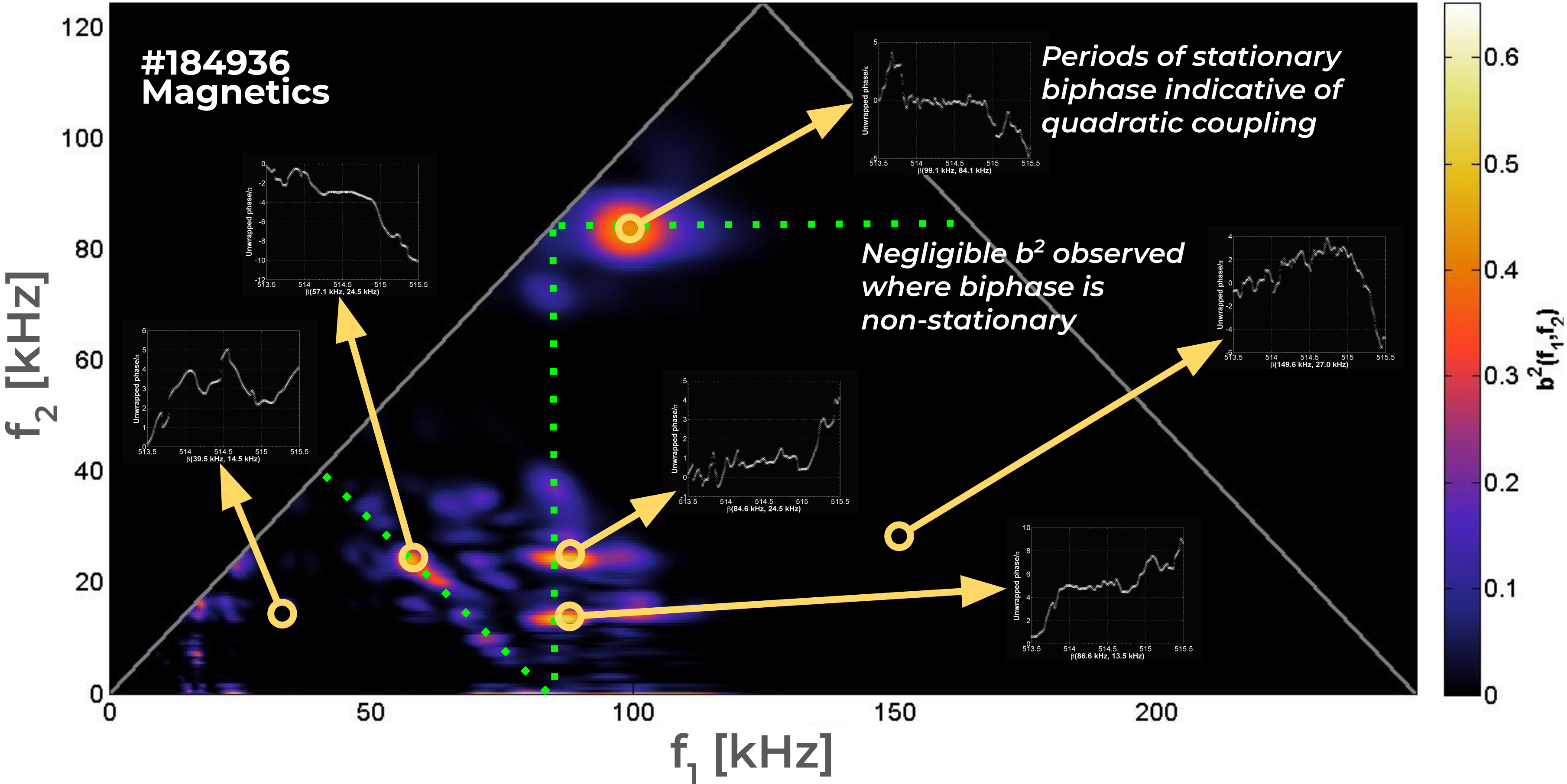
Wavelet-based bicoherence enables highly time- and frequency-resolved assessment of phase-coherency



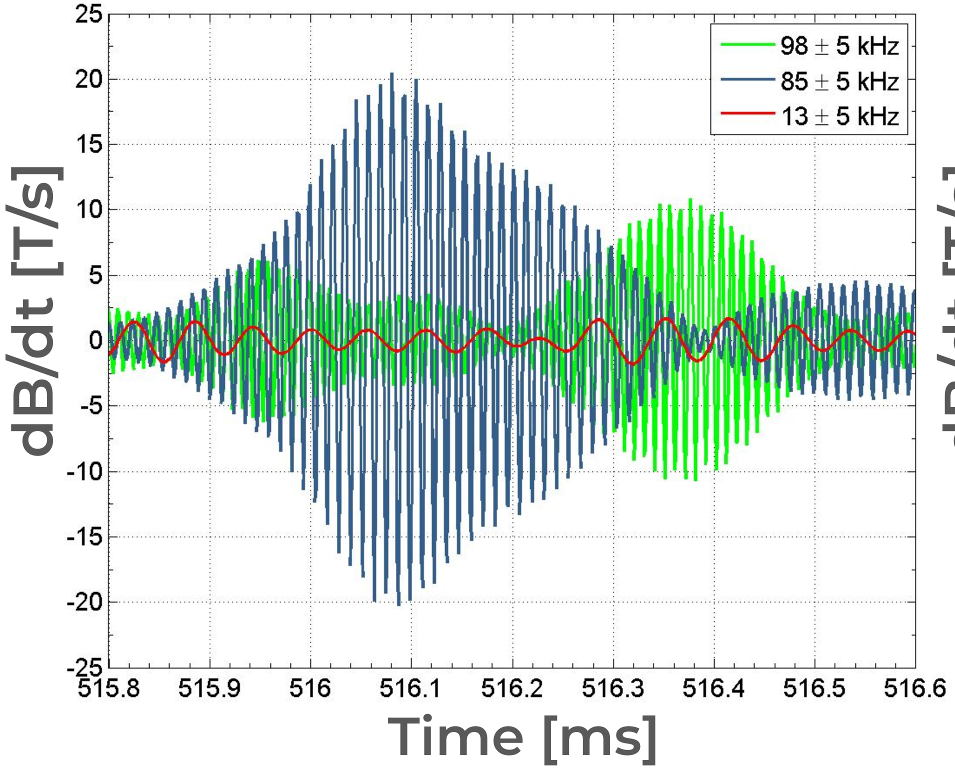
Mode-mode interaction evinced by coherent sum and difference frequencies

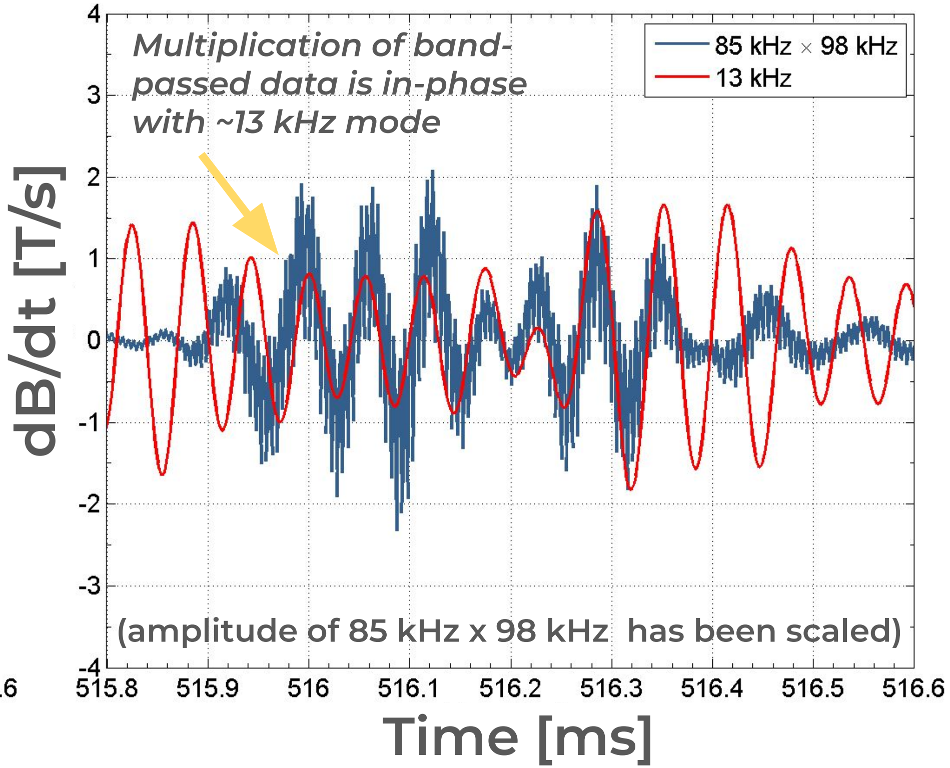


Biphase evolution is consistent with nonlinear 3-wave coupling

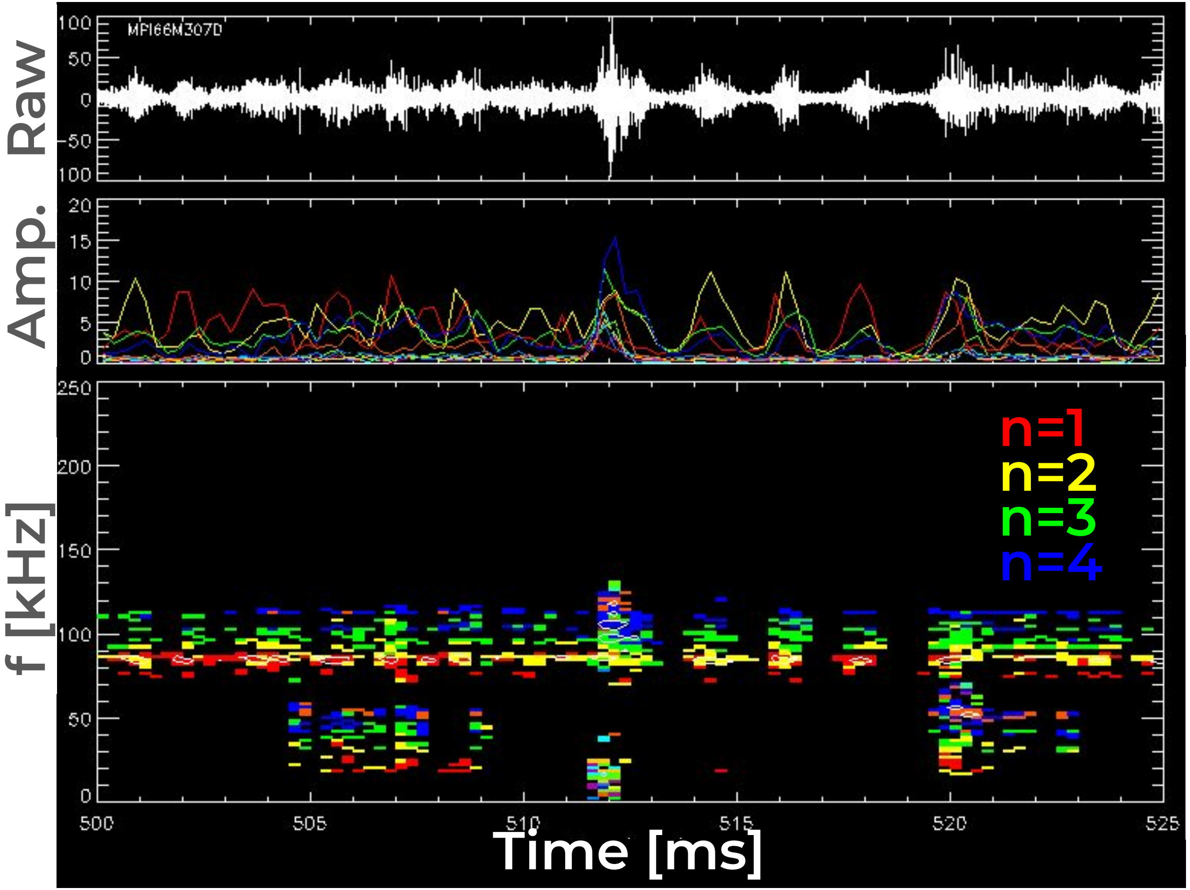


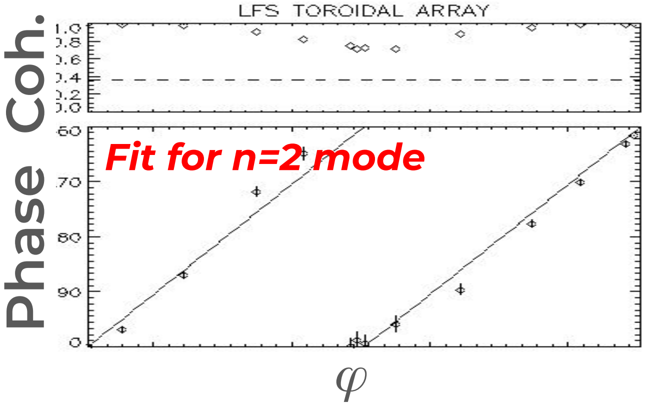
Interpretation supported by correlation between beat dynamics and low-frequency fluctuations

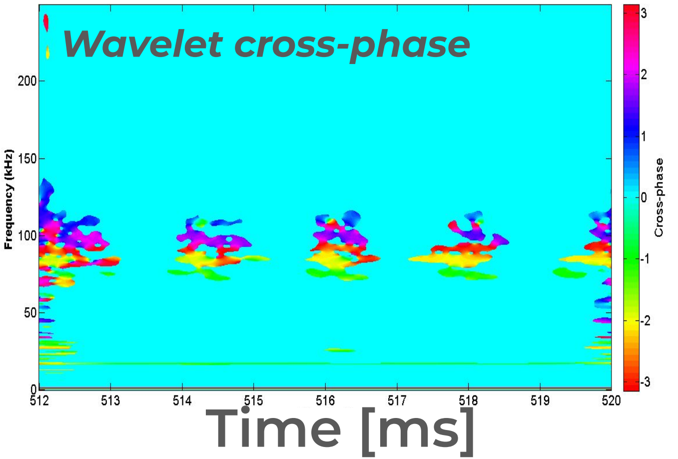




Mode identification benchmarked by cross-phase analysis[6]



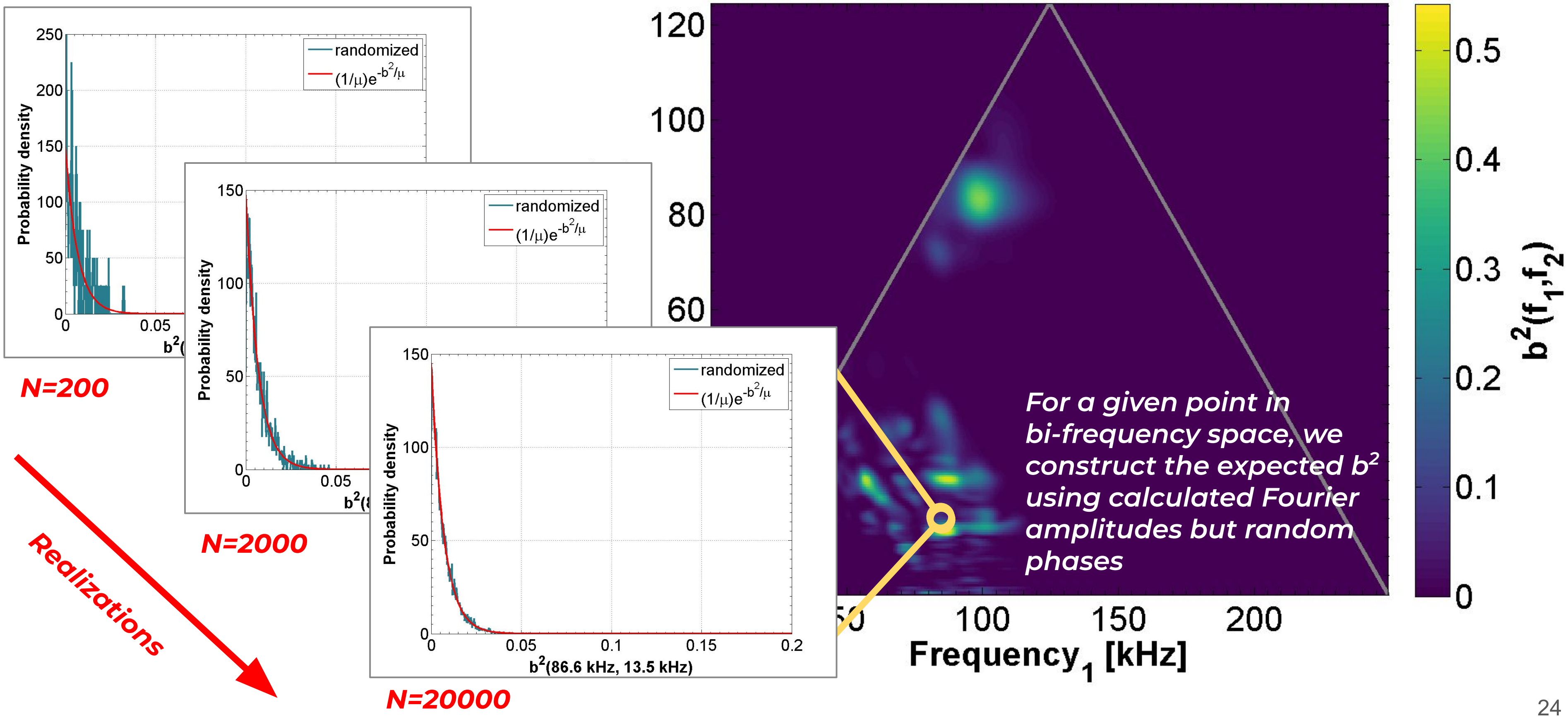




[6] Strait, E. J. *Magnetic diagnostic system of the DIII-D tokamak*, Review of scientific instruments 77(2): 023502 (2006); https://doi.org/10.1063/1.2166493

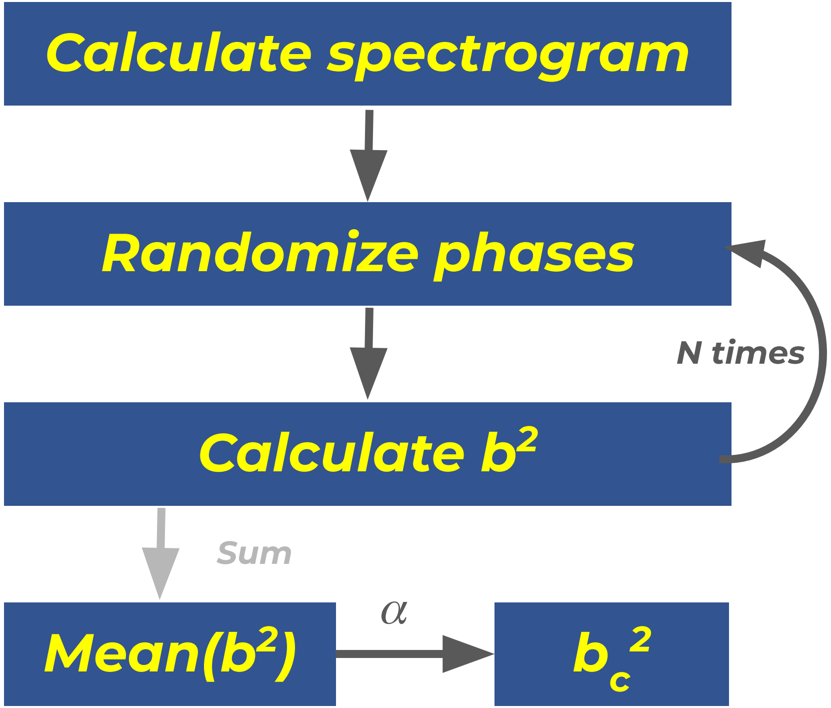
Assessment of noise floor

Expected value of bicoherence is typically well-approximated by exponential distribution



For a given point in bi-frequency space, we construct the expected b^2 using calculated Fourier amplitudes but random phases

Confidence interval may be derived using quantile[7]



Modelled distribution given by:

$$\mathrm{PDF}(b^2) = \frac{e^{-b^2/\mu}}{\mu}$$

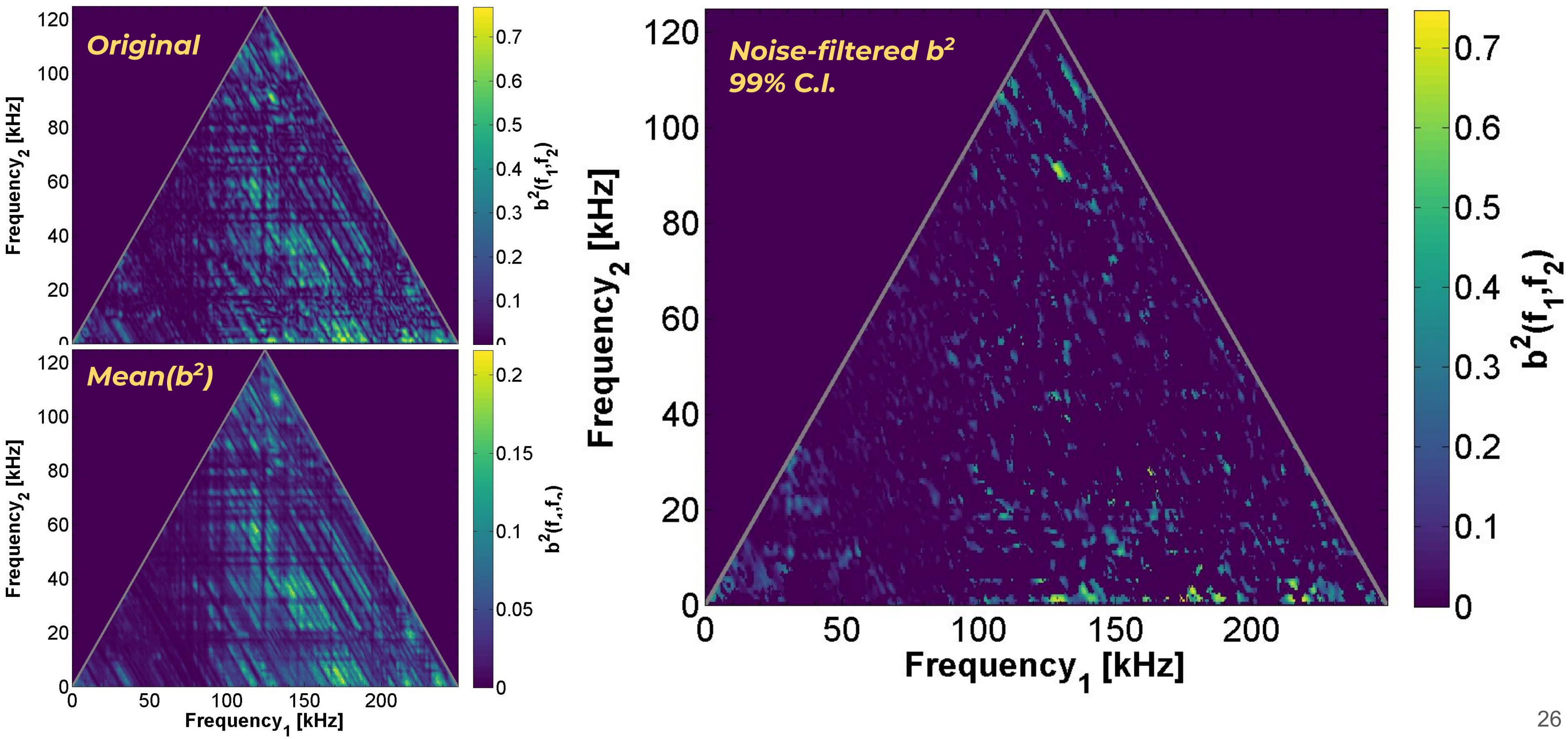
where μ is mean of random-phase realizations

Critical value of bicoherence for confidence level $0 < \alpha < 1$:

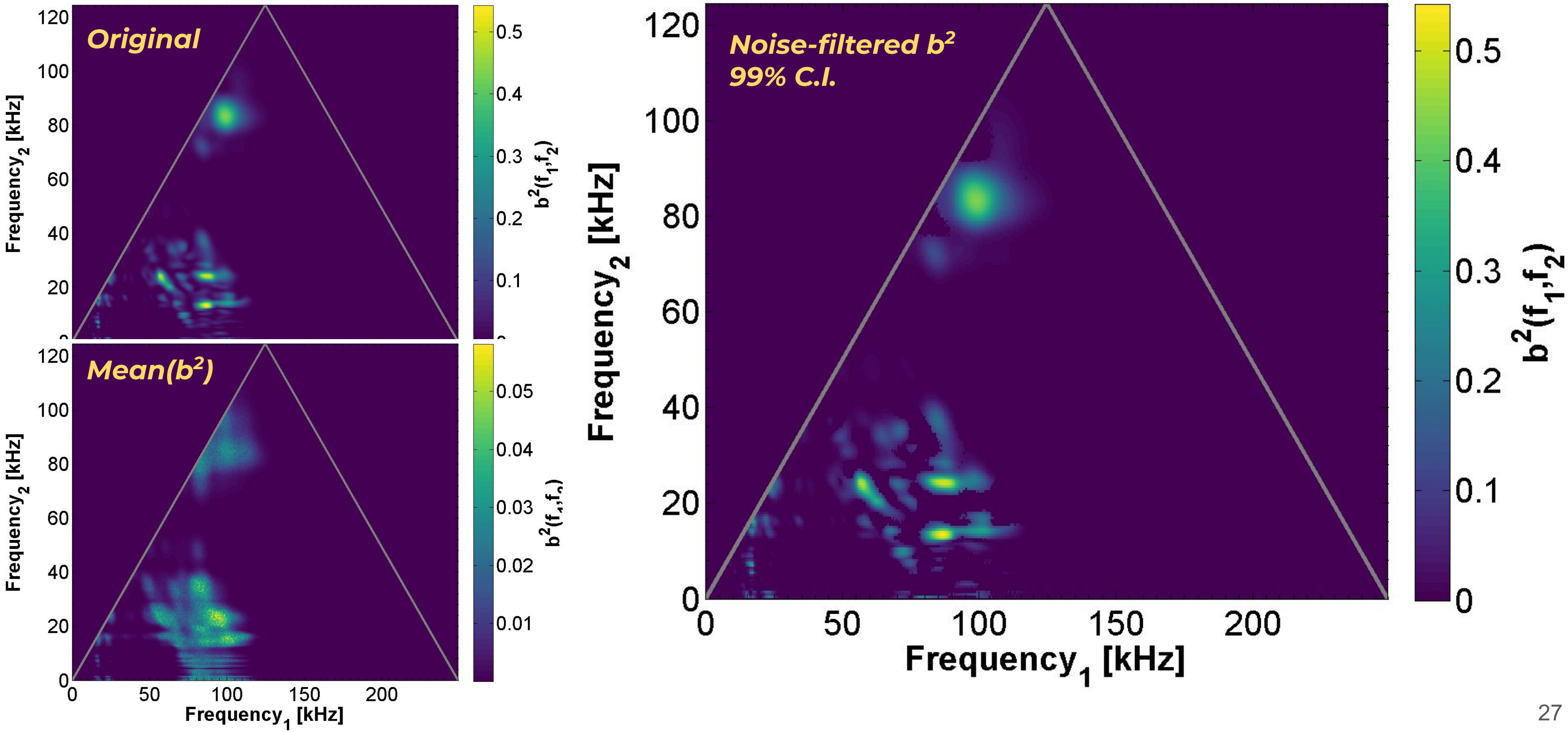
$$b_c^2 = -\mu \log(1-\alpha)$$

[7] Poloskei, P. et al., *Bicoherence analysis of nonstationary and nonlinear processes.* (2018) arXiv:1811.02973

Technique provides robust filtering of spurious bicoherence for non-stationary signals



Consistent phase-coherency is unlikely to be discarded



What's next?

Priorities for 2023

Use FAR3d and TRANSP simulations to provide insight into wave-wave and wave-particle interactions

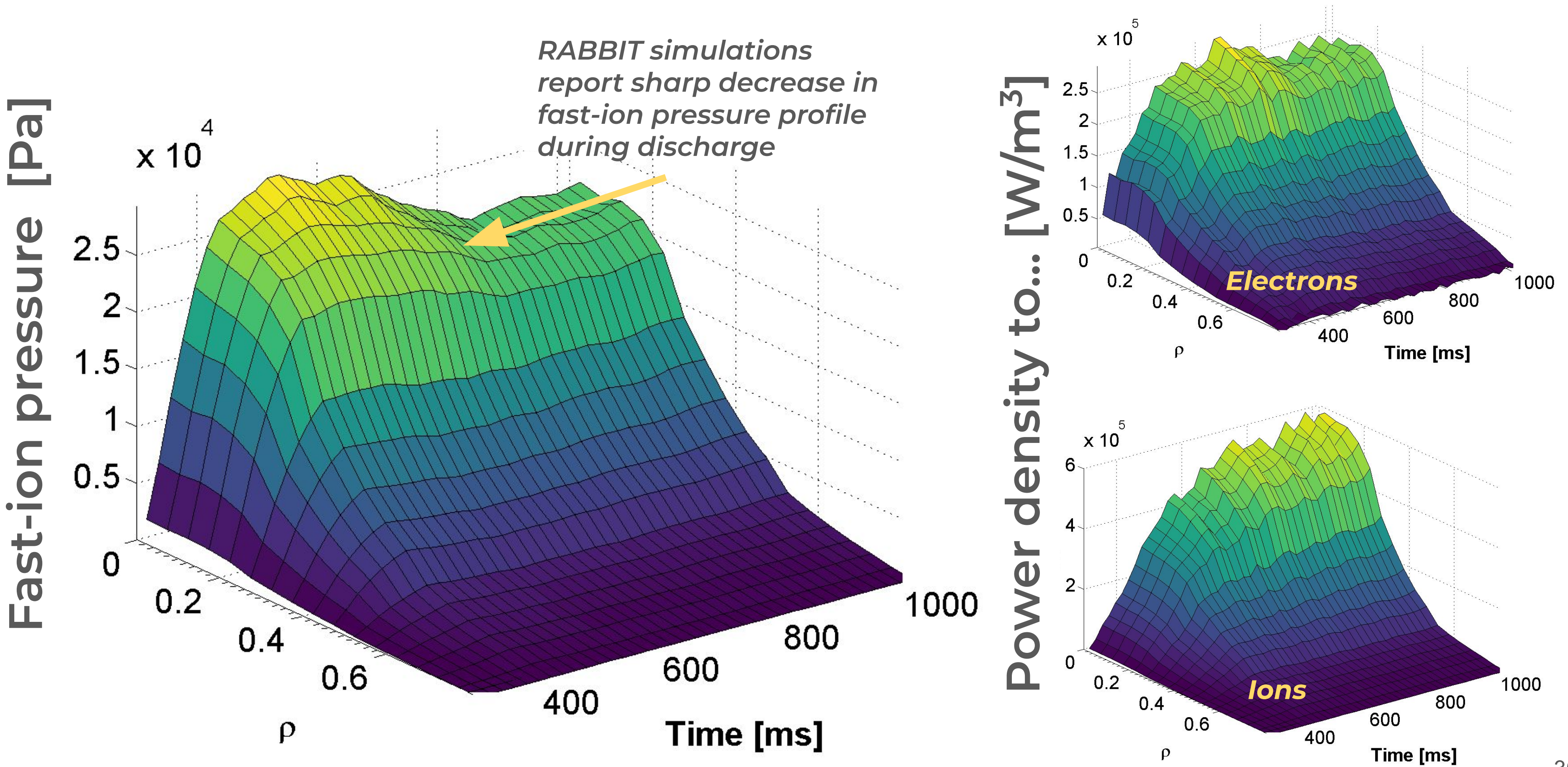
Correlate fluctuations in density and magnetic field with perturbations in fast-ion distribution function (*e.g.*, FILD fluctuation analysis)

Quantify role of nonlinear coupling and energy transfer in mediating saturated amplitude of TAEs

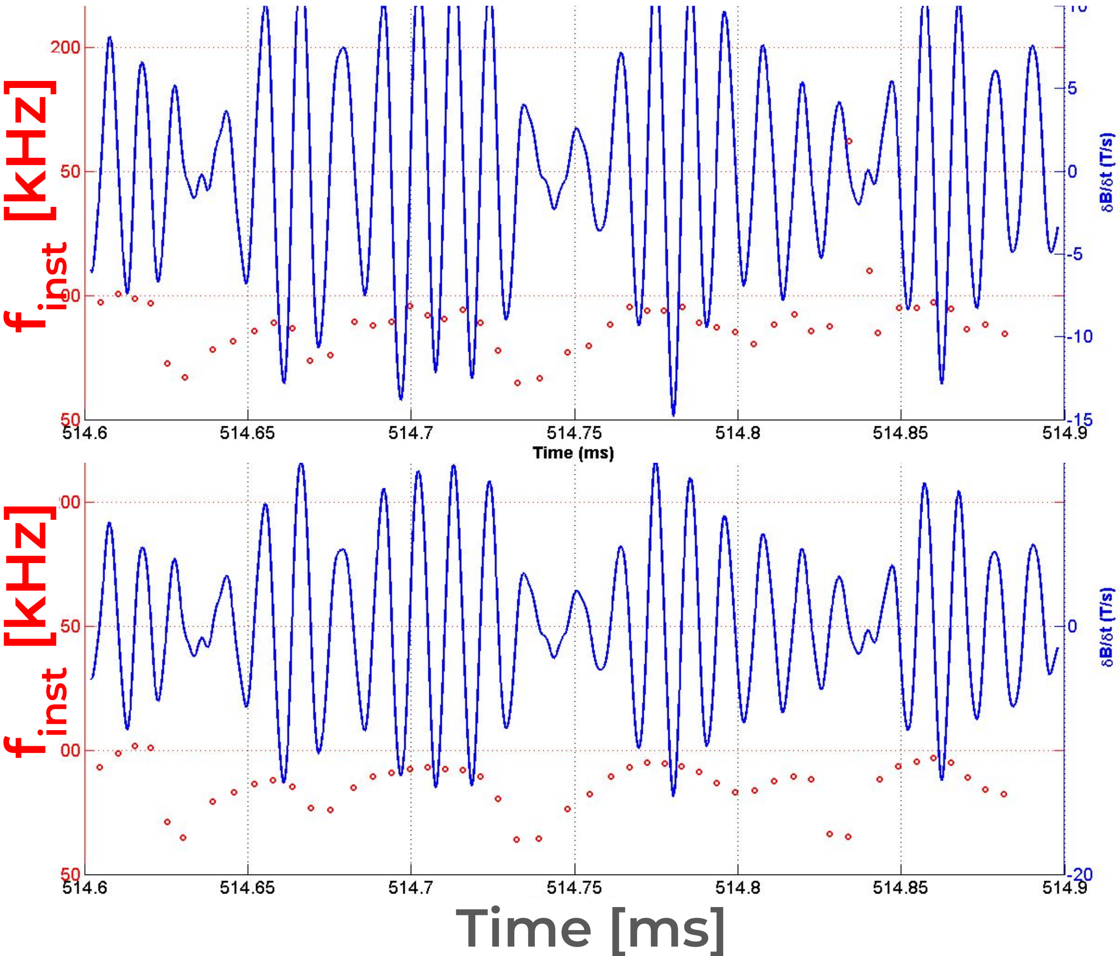
Assess efficacy of Berk-Breizman model to explain observed AM/FM, or other nonlinear effects

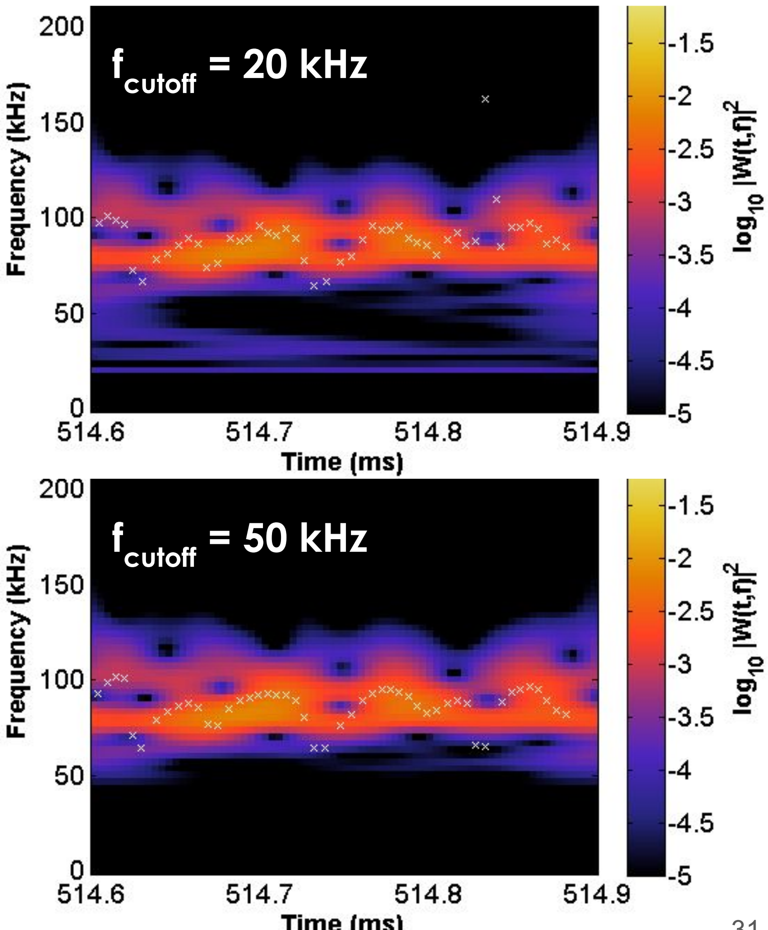
Develop “flowchart”

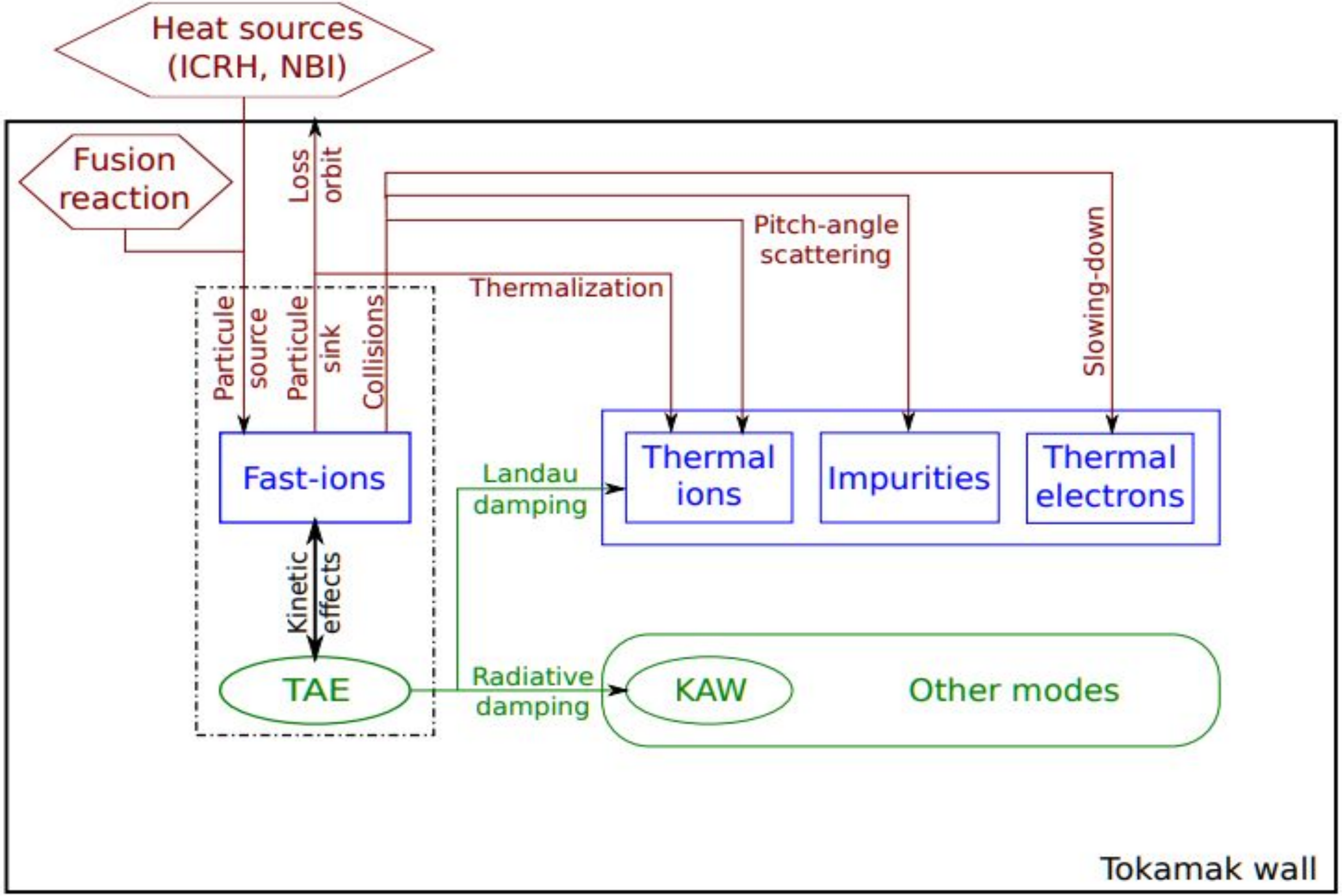
Mechanism of saturation is under investigation



Frequency modulation likely in HP-filtered data







[8] Lesur, M., *The Berk-Breizman Model as a Paradigm for Energetic Particle-driven Alfven Eigenmodes*, Ecole Polytechnique (2010)

Appendix: Bicoherence primer

Bispectrum is generalization of power spectrum[9]

Fourier transform of auto/cross-correlation = power/cross-spectrum

Bispectrum = higher order transform; identifies nonlinear interactions via

$$\mathcal{B}_{x_1 x_2 x_3}(f_1, f_2) = \left\langle \hat{x}_1(f_1)\, \hat{x}_2(f_2)\, \overline{\hat{x}_3(f_1 + f_2)} \right\rangle$$

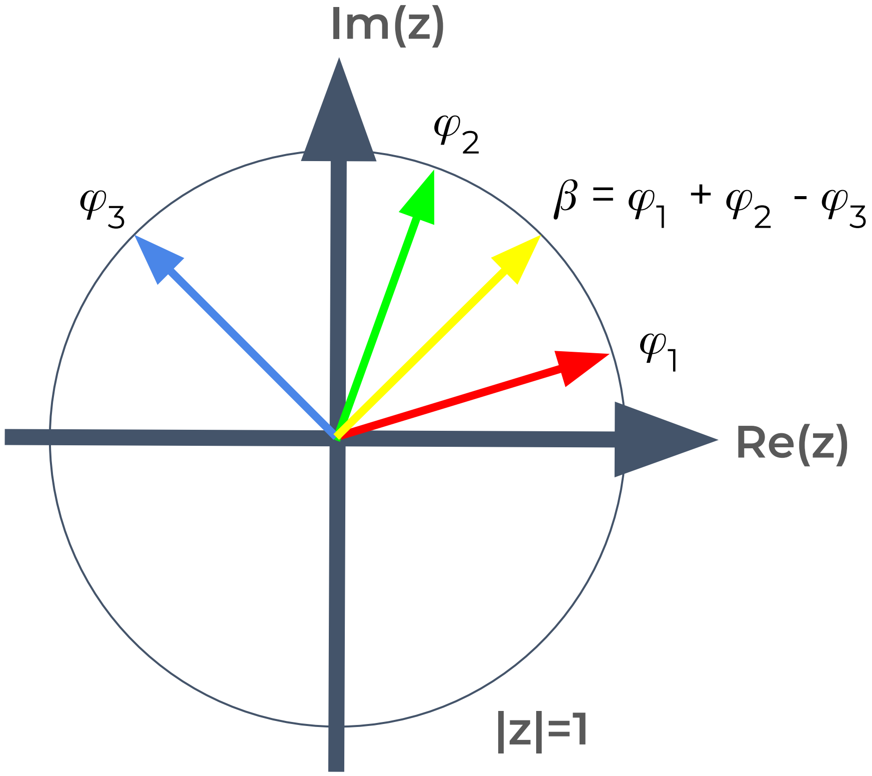
Auto/cross-bicoherence = normalized auto/cross-bispectrum

$$b^2_{x_1 x_2 x_3}(f_1, f_2) = \frac{\left|\mathcal{B}_{x_1 x_2 x_3}(f_1, f_2)\right|^2}{\left\langle \left|\hat{x}_1(f_1)\hat{x}_2(f_2)\right|^2 \right\rangle \left\langle \left|\hat{x}_3\left(f_1 + f_2\right)\right|^2 \right\rangle}$$

Detects complementary phase relationships between frequency triples

[9] Kim & Powers, *Digital bispectral analysis of self-excited fluctuation spectra,* Phys. Fluids **21**, 1452 (1978); https://doi.org/10.1063/1.862365

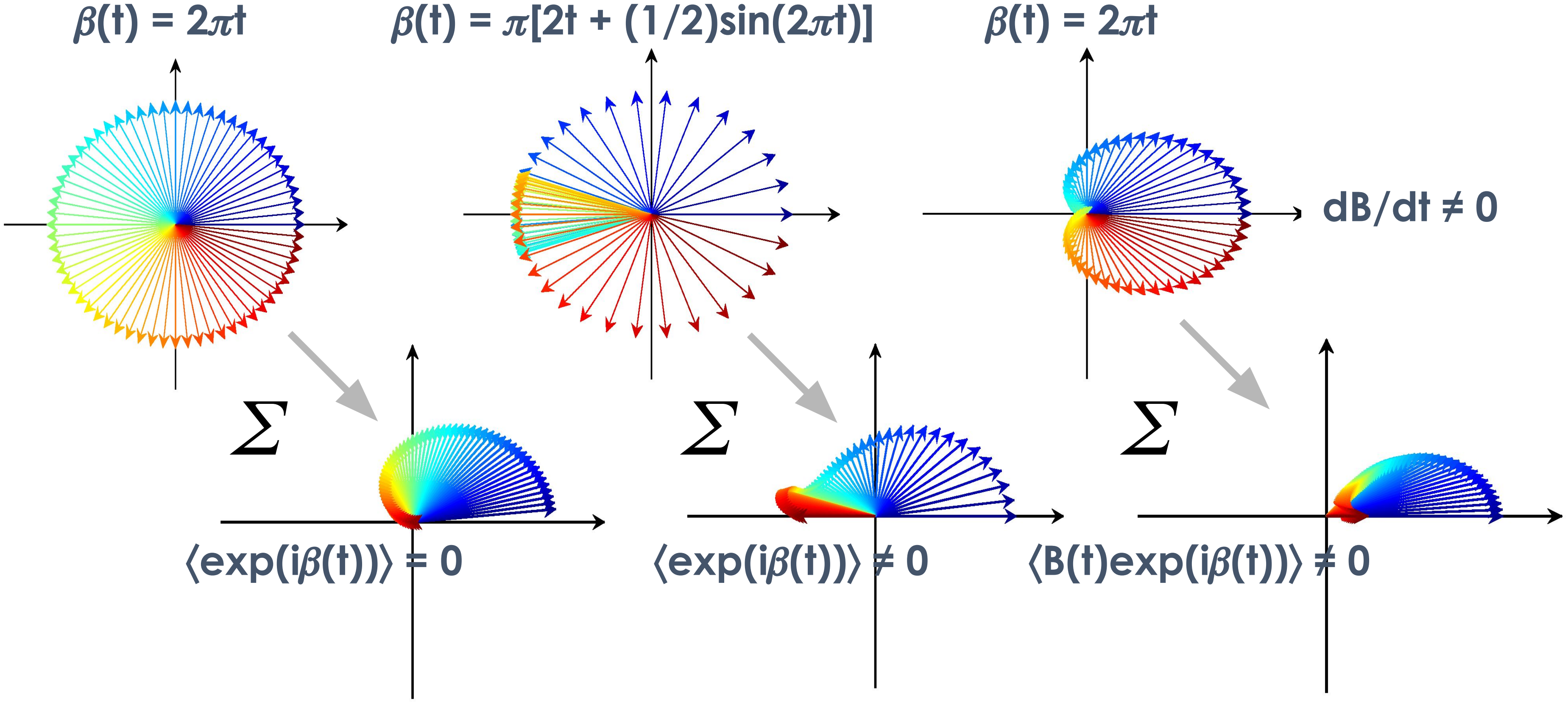
Bicoherence determined by *phase-coherency*



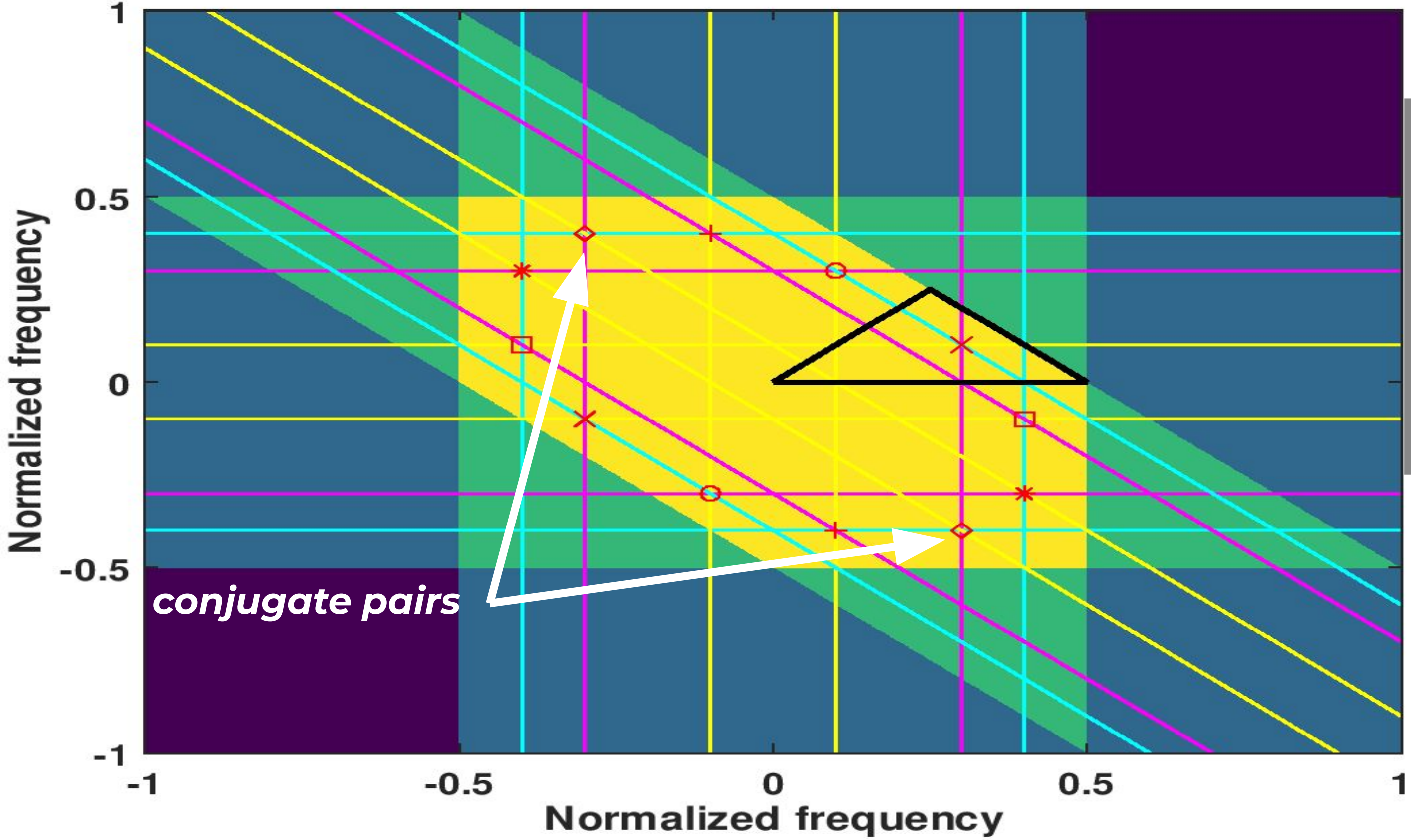
$$\mathcal{B}_{x_1x_2x_3}(f_1, f_2) \propto \left\langle e^{i\beta(f_1,f_2,\tau)} \right\rangle$$

- **When Fourier amplitudes are slowly varying in time, B(f_1, f_2) depends entirely on the dynamics of biphase β**
- Crucially, the bispectrum will tend to null when β is random or linear in time
- **A static biphase thus corresponds to nonzero values of bicoherence**
- Oscillatory biphase does *not* generally lead to vanishing bicoherence

Phase and amplitude modulation require careful interpretation



Bi-frequency space is 12-fold degenerate for auto-bicoherence analyses



$f_3 = 0.4\ f_s$

$f_2 = 0.3\ f_s$

$f_1 = 0.1\ f_s$

Thanks for your time!

email : gariggs@mix.wvu.edu